Penalized Maximum-likelihood PET Image Reconstruction for Lesion Detection

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Outline

• Introduction and motivation
• Image reconstruction (PML)
• Task-based image quality
• Optimizing PML reconstruction
  • Penalty design
  • Resolution modeling
• Extension to dynamic PET
Positron Emission Tomography

- Use radioactive tracer to imaging function of living body
- Based on detection of two 511 keV photons produced in positron annihilation
Motivation

• PET is widely used in oncology
  • Increasingly being used for staging and treatment monitoring
  • Challenging to detect small tumors (< 10 mm)

• Numerous efforts have been focused on improving detection of small tumors by developing
  • New PET tracers
  • New PET scanners: time-of-flight, dedicated systems, etc.
  • Optimized image reconstruction methods (our focus)
Statistical Reconstruction

- Noise-free emission data $\tilde{y}$ can be modeled as
  \[ \tilde{y} = Px + r \]
- Measured data $y$ follows Poisson distribution
  \[ Pr(y|\tilde{y}) = \prod_{i=1}^{\tilde{y}} \frac{\tilde{y}_i^{y_i} e^{-\tilde{y}_i}}{y_i!} \]
- Log-likelihood function
  \[ L(y|x) = \sum_i (y_i \log[Px + r]_i - [Px + r]_i) \]
- Maximum-likelihood reconstruction
  \[ \hat{x} = \arg \max_{x \geq 0} L(y|x) \]
- Noisy solution at convergence
PML Reconstruction

• Penalized maximum-likelihood solution

\[ \hat{x}(y) = \arg \max_{x \geq 0} [L(y|x) - \beta \phi(x)] \]

• Quadratic penalty function

\[ \phi(x) = \sum_{j=1}^{N} \sum_{l \in \mathcal{N}_j} \gamma_{jl} (x_j - x_l)^2 = x^t Rx \]

• Challenges:
  • Selection of penalty parameters \( \beta \), or more sophisticatedly, the individual \( \{y\} \)
  • System modeling \( P \)
Effect of $\beta$

Real patient whole-body FDG PET images
@ Toshiba Celesteion PET/CT scanner

Smaller $\beta$  Larger $\beta$

Courtesy of Dr. Jian Zhou @ Toshiba
Previous Research

• Quadratic penalty parameters ($\beta$, or \{\gamma\}) has been optimized for different tasks:
  • Achieve uniform resolution  
    (Stayman and Fessler 2000, 2004, Qi and Leahy 2000)
  • Optimize local contrast to noise ratio  (Qi and Leahy 1999)
  • Improve lesion detectability in 2D  (Qi and Huesman 2006)

• **Goal**: to optimize penalty function for lesion detection in fully 3D
Lesion Detection

- Goal standard for measuring lesion detectability is human ROC study
Lesion Detection

• Goal standard for measuring lesion detectability is human ROC study

• Computer observers are used to reduce cost and provide possibility for theoretical analysis
Computer Observer

• Test statistics

\[ \hat{\mathcal{X}} \rightarrow \text{Computer} \]

\[ \eta(\hat{x}) \begin{cases} < \eta_c \rightarrow \text{Normal (H}_0) \\ > \eta_c \rightarrow \text{Abnormal (H}_1) \end{cases} \]

• Figure of merit (FOM)

\[
\text{SNR}^2[\eta(\hat{x})] = \frac{(E[\eta(\hat{x})|H_1] - E[\eta(\hat{x})|H_0])^2}{(\text{var}[\eta(\hat{x})|H_1] + \text{var}[\eta(\hat{x})|H_0])/2}
\]

\[
\text{AUC} = \frac{1}{2}[1 + \text{erf}(\frac{\text{SNR}}{2})]
\]

• Channelized Hotelling observer (CHO)

• Good correlation with human performance

(Yao and Barrett 1992, Abbey et al 1996)
2D CHO

Three difference of Gaussian (DOG) channels $U$

2D image $\hat{x}$

Channel outputs $U\hat{x}$

Internal noise $\eta$
- zero mean
- covariance $K_\eta$

Test statistics $z'U'K^{-1}(U\hat{x} + n)$

( $z$ : mean reconstructed lesion profile)

Hotelling Observer

$U\hat{x} + n$
Single-slice CHO

"3D" channels
$U$

3D image
$\hat{x}$

Internal noise $n$
- zero mean
- covariance $K_n$

Channel outputs
$U\hat{x}$

Test statistics
$z'U'K^{-1}(U\hat{x} + n)$

( $Z$ : mean reconstructed lesion profile)

Hotelling Observer

$U\hat{x} + n$
Multi-slice CHO

“3D” channels $U$

3D image $\hat{x}$

Channel outputs $U\hat{x}$

Internal noise $n$
- zero mean
- covariance $K_n$

Test statistics $z'U'K^{-1}(U\hat{x} + n)$

( $Z$ : mean reconstructed lesion profile)

Hotelling Observer

$U\hat{x} + n$
Multi-view CHO

\[ FOM : SNR^2 = z'U'K^{-1}Uz \]

Test statistics \( z'U'K^{-1}(U\hat{x} + n) \)

( \( z \): mean reconstructed lesion profile)

Channel outputs \( U\hat{x} + n \)

Internal noise \( n \)
- zero mean
- covariance \( K_n \)

\[ \text{Hotelling Observer} \]

\( \hat{x} \)

\( U \)

\( \text{3D image} \)

\( \text{3D channels} \)

\( U \)

\( \text{3D image} \)

\( \text{3D } \)
mvCHO SNR of PML

• For lesion at a given location, we derived a theoretical expression for $U_z$ and $K$, using Taylor series and locally shift invariant approximation (Qi 2004)

$$U_z \approx \mathcal{F}\{U\}\text{diag}\left[\frac{\mathcal{F}\{P'\text{diag}\left[\frac{1}{y}\right]Pe_j\} \odot \mathcal{F}\{f_l\}}{\mathcal{F}\{P'\text{diag}\left[\frac{1}{y}\right]Pe_j\} + \beta\mathcal{F}\{Re_j\}}\right]$$

$$K \approx \mathcal{F}\{U\}\text{diag}\left[\frac{\mathcal{F}\{P'\text{diag}\left[\frac{1}{y}\right]Pe_j\}}{\left(\mathcal{F}\{P'\text{diag}\left[\frac{1}{y}\right]Pe_j\} + \beta\mathcal{F}\{Re_j\}\right)^2}\right] \mathcal{F}\{U\}' + K_N$$

- $\mathcal{F}\{\cdot\}$: Fourier transform; $\odot$: element-by-element multiplication
- $f_l$: mean lesion profile; $e_j$: unit vector

• Lesion detectability

$$\text{SNR}^2_{CHO}(\beta\mathcal{F}\{Re_j\}) = z'U'K^{-1}Uz$$
Objective: to maximize $\text{SNR}^2(\beta \mathcal{F}\{\text{Re}_j\})$

Parameterization of penalty kernel:

$\text{Re}_j = \text{Ker}_j = \sum_l \gamma_{jl}b_{jl}$  (Stayman et al 2000)

Equivalent to finding the optimum weights $\gamma$:

$\hat{\gamma} = \arg \max_{\gamma} \text{SNR}^2(\gamma)$

We used 92 nearest neighboring voxels ($l = 46$) in 3D

The penalty weights were computed for a set of preselected voxels on a course grid and the nearest neighbor interpolation was used to form the overall penalty function
Procedure of Penalty Design

1: For a given sinogram $\mathbf{y}$, approximate $\text{diag}[\frac{1}{\mathbf{y}}]$ by $\text{diag}[\frac{1}{\mathbf{y}+1}]$.

2: Precompute the Fourier transform of the channel functions, $\mathcal{F}\{\mathbf{U}\}$.

3: Precompute the Fourier transform of each pairwise penalty kernel $\mathcal{F}\{\mathbf{b}_{jl}\}$.

4: for each voxel $j$ on the coarse grid do

5: Simulate a small lesion $\mathbf{f}_l$ as a hot spot at voxel $j$, and compute its Fourier transform $\mathcal{F}\{\mathbf{f}_l\}$.

6: Forward project the $j$th unit vector $\mathbf{e}_j$ by the system matrix $\mathbf{P}$ to obtain $\mathbf{P}\mathbf{e}_j$.

7: Perform a weighted back projection to get $\mathbf{P}'\text{diag}[\frac{1}{\mathbf{y}+1}]\mathbf{P}\mathbf{e}_j$, and compute its Fourier transform $\mathcal{F}\{\mathbf{P}'\text{diag}[\frac{1}{\mathbf{y}+1}]\mathbf{P}\mathbf{e}_j\}$.

8: Use Matlab function “fmincon” to estimate the weighting factors $\gamma_{jl}$ that maximize $\text{SNR}_{\text{mvCHO}}^2$.

9: end for

10: Assign the weighting factors to other voxels using nearest neighbor interpolation.
Outline

• Task-based PML reconstruction
  • Penalty design
    • Validation using computer-based simulation
    • Validation using real data
  • Resolution modeling
    • Validation using computer-based simulation
    • Validation using real data
• Extension to dynamic PET
Simulation Method

• GE DST whole-body scanner:
  • FOV: axial 157 mm, transaxial 700 mm
  • Crystals size: 6.34 x 6.34 x 30 mm

• Digital phantom

• Simulated tumors:
  • Five small lesions with diameter of 3 mm
  • A large lesion with diameter of 8 mm
  • Activity ratio 2.2:1

• Phantom with and without tumor were forward projected

• Independent Poisson noise was added to generate 200 independent noisy realizations, each with 100M total counts
Comparison of 3D CHOs

- Efficiency of four 3D CHOs for lesion detection

- Multi-view CHO > multi-slice CHO, single-slice CHO
- For small lesion, multi-view CHO ≈ multi-slice multi-view CHO
Detection Performance

- Lesion detectability of PML reconstructions using 1\textsuperscript{st} order quadratic penalty and proposed penalty at two representative locations:

<table>
<thead>
<tr>
<th>Location 1</th>
<th>Location 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR of theoretical (curves) and Monte Carlo results (&quot;x&quot; &amp; &quot;o&quot;)</td>
<td>SNR of theoretical (curves) and Monte Carlo results (&quot;x&quot; &amp; &quot;o&quot;)</td>
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</table>
Sample Reconstruction

1\textsuperscript{st} order quadratic penalty

Proposed penalty

Transverse       Coronal       Sagittal
Sample Reconstruction

Transverse  Coronal  Sagittal

1st order quadratic penalty

Proposed penalty
Human Observer Test

- To verify the numerical observer results, we perform two alternative forced choice (2AFC) experiments.

**Test process**

\[ PC = \frac{\text{# of correct choice}}{\text{# of total test pairs}} \]

- The resulting percent correct (PC) can be converted to SNR:

\[ SNR = 2\text{erf}^{-1}(2PC - 1) \quad (\text{Burgess 1995}) \]
Human Observer Results

- SNR of theoretical (curves) and human observer results ("x" and "o")

Observer 1

Observer 2

Location 1

Location 2

Proposed penalty

1st order quadratic penalty

\[ \text{SNR} \]

\[ \text{log}_{10}(\beta) \]
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Clinical Implementation

CT Data Acquisition → CT Image Reconstruction

PET Data Acquisition → Prompts
- Normalization
- Attenuation Correction
- Random Correction
- Scatter Correction

Penalty Design

PML Image Reconstruction

GE DST Scanner
Real Data Evaluation

• Gold standard (perfect knowledge of the presence and location of each lesion)
• Lesion-free patient background
  • A 60-minute PET scan of a female patient
  • 5 mCi FDG injection
• The FOV covered the heart, breasts, and part of the lungs and liver
• The last 45 minutes data were summed to create a high-count sinogram with ~800M events (9× of a normal scan)
Lesion-Present Data

- A Na-22 point source was scanned in air at 27 locations.
  - Attenuated by the patient body
  - Added to the patient sinogram
  - 7 implausible positions were excluded

- Sample reconstruction with a superimposed lesion in the liver

- Independent Poisson noise was introduced to generate 200 independent noisy realizations, each with 90M total counts, mimicking a 5-minute scan
Image Reconstruction

- Reconstruction algorithm:
  - PML with the 1st order quadratic penalty
  - PML with the optimized penalty
- Image matrix size: $192 \times 192 \times 47$
- Voxel size: $3.64 \times 3.64 \times 3.27$ mm
- Randoms, scatters, and normalization factors were estimated from the patient data using a manufacturer provided software and included in the reconstruction
- Both reconstruction methods used the same forward/backword projectors and correction factors
Detection Performance

• Lesion detectability of PML reconstructions at two representative locations

Location 1

Location 2

SNR of theoretical (curves) and Monte Carlo results ("x" & "o")
Human Observer Results

- One human observer 2AFC experiment

SNR of theoretical (curves) and human observer results ("x" & "o")
Human Observer Results

- Human observer results at all 20 tumor locations
- The proposed penalty improves the SNR by up to 15%
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    • Validation using computer-based simulation
    • Validation using real data

• Extension to dynamic PET
Resolution Modeling

- Without the normalization and attenuation, the system matrix:

\[ P = P_{\text{det.blur}} P_{\text{geom}} P_{\text{positron}} \]

(Qi et al 1998)

- \( P_{\text{geom}} \): geometrical projection matrix
- \( P_{\text{positron}} \): positron range
- \( P_{\text{det.blur}} \): photon acolinearity, crystal penetration, inter-crystal scatter

- Point source example

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Credit: Courtesy of Dr. Michel Tohme @ GE
Image-space RM

- In practice, the system matrix is usually reduced to:

  **Sinogram space RM**
  \[ P_r = SG \]
  
  **Image space RM**
  \[ P_r = GB \]

  - *S*: sinogram space blurring matrix;
  - *B*: image space blurring matrix
  - *G*: geometrical projection matrix

- **Advantage** of image-space RM: suitable for list-mode reconstruction

- Blurring matrix *B* can be estimated using measured point spread function (PSF)
  - Measured PSF may overestimate true PSF (Watson 2011)
  - May not be optimal for detection task (Rahmim and Tang 2013, Ashrafinia et al. 2014)
CHO SNR vs RM

- FOM: \( \text{SNR}_{\text{CHO}}^2 = z'U'K^{-1}Uz \)
- Extend our theoretical expression of \( Uz \) and \( K \) to evaluate the effect of RM

\[
Uz \approx \mathcal{F}\{U\}\text{diag}
\left[ \begin{array}{c}
\frac{\mathcal{F}\{G'D'\text{diag}[\frac{1}{y}]P_m f_l\} \odot \mathcal{F}\{Be_j\}}{\mathcal{F}\{G'D'\text{diag}[\frac{1}{y}]DGe_j\} \odot \mathcal{F}\{Be_j\}^2 + \beta \mathcal{F}\{Re_j\}} \\
\end{array} \right]
\]

\[
K \approx \mathcal{F}\{U\}\text{diag}
\left[ \begin{array}{c}
\frac{\mathcal{F}\{G'D'\text{diag}[\frac{1}{y}]DGe_j\} \odot \mathcal{F}\{Be_j\}^2}{(\mathcal{F}\{G'D'\text{diag}[\frac{1}{y}]DGe_j\} \odot \mathcal{F}\{Be_j\}^2 + \beta \mathcal{F}\{Re_j\})^2} \\
\end{array} \right] \mathcal{F}\{U\}' + K_N
\]

\( \mathcal{F}\{\cdot\} \): Fourier transform; \( \odot \): element-by-element multiplication

\( f_l \): mean lesion profile; \( e_j \): unit vector

- Lesion detectability:

\[
\text{SNR}_{\text{CHO}}^2(Be_j, \beta) = z'U'K^{-1}Uz
\]
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Simulation Method

• GE DST scanner geometry
• 2D digital phantom
  • 200 x 200 pixels with size 3.2 x 3.2 mm
  • A small tumor with contrast 5:1
• Three scanner PSFs were simulated using a Gaussian function with different FWHM $\sigma_t$:
  - $\sigma_t = 4.8$ mm
  - $\sigma_t = 6.4$ mm
  - $\sigma_t = 9.6$ mm
RM used in reconstruction was a shift-invariant Gaussian function with FWHM $\sigma_r$ (0-22 mm)

$$\text{SNR}^2_{\text{CHO}}(\sigma_r) = \max_{\beta} \text{SNR}^2_{\text{CHO}}\left(B(\sigma_r)e_j, \beta\right)$$

$\sigma_t = 4.8$ mm

$\sigma_t = 6.4$ mm

$\sigma_t = 9.6$ mm

<table>
<thead>
<tr>
<th>True $\sigma_t$</th>
<th>Optimal $\sigma_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8 mm $\geq$ 0 mm</td>
<td></td>
</tr>
<tr>
<td>6.4 mm $\equiv$ 6.4 mm</td>
<td></td>
</tr>
<tr>
<td>9.6 mm $\leq$ 17.6 mm</td>
<td></td>
</tr>
</tbody>
</table>
Monte Carlo SNR was computed from 200 noisy realizations, each with 1M total counts.

\[ \sigma_t = 4.8 \text{ mm} \]

\[ \sigma_t = 9.6 \text{ mm} \]

SNR of theoretical (curves) and Monte Carlo results ("x", "o", "★")
Outline

• Task-based PML reconstruction
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• Resolution modeling
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• Extension to dynamic PET
RM for Real Data

- Patient data with a superimposed artificial lesion
- Resolution modeling
  - Use a mixture of two Gaussian functions to fit point source reconstruction
  - The fitted kernel scaled by a factor $\alpha \in [0 - 1.5]$ was used as RM in reconstruction

$\alpha = 0.8$  \hspace{1cm}  $\alpha = 1$  \hspace{1cm}  $\alpha = 1.2$
Lesion Detectability vs $\alpha$

In both cases, an under-estimated resolution kernel ($\alpha = 0.8$) resulted in the highest SNR.
Monte Carlo SNR was computed from 200 noisy realizations, each with around 1M total counts, mimicking a 5-minute scan.

SNR as a function of $\beta$:

\[ \hat{\alpha} = \arg \max_{\alpha \geq 0} \text{SNR}_{\text{CHO}}^2(\alpha) \]

Task-based RM for Real Data

- Task-based RM
- Penalty design
- No RM

SNR of theoretical (curves) and Monte Carlo results ("x", "o", "★")
Human Observer

- One human observer 2AFC experiment

![Bar chart showing performance metrics for different conditions and locations.](chart.png)
Outline

• Task-based PML reconstruction
  • Penalty design
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    • Validation using computer-based simulation
    • Validation using real data
• Extension to dynamic PET
Dynamic PET

- Standard static PET
  - Scan performed around 60 min post injection
  - Tracer concentration at a single time point

- Dynamic PET
  - Follow tracer concentration over a period of time
  - Time activity curve (TAC)

- Tracer kinetic model
  - Extract physiological parameters from TAC

![TAC Graph](image-url)
**Patlak Plot**

**Blood input function** $C_p(t)$

\[
\frac{c(t)}{C_p(t)} = \kappa_i \frac{\int_0^t C_p(\tau) d\tau}{C_p(t)} + b
\]

\(\kappa_i\) : influx rate, and can be used for lesion detection (Kenny et al. 2005)

**Tissue TAC** $c(t)$

(Patlak and Blasberg 1985)
Dynamic PET Reconstruction

Data Acquisition

Dynamic Sinograms \( \{y\} \)

Frame-by-frame Reconstruction

Dynamic Images \( \{x\} \)

Tracer Kinetic Modeling

Time Activity Curve (TAC)

Activity

Scan time (min)

Parametric Images \( \{\kappa_i, b\} \)

Pixel-wise Fitting

Direct Parametric Imaging

Direct method

Indirect method
Data Model

- Dynamic image:

\[ x = [x'_1 \ x'_2 \ldots \ x'_T]' = \left( A \bigotimes I_N \right) \begin{bmatrix} \kappa_i \\ b \end{bmatrix}, \ A = \begin{bmatrix} \bar{S}_p(1) & \bar{C}_p(1) \\ \bar{S}_p(2) & \bar{C}_p(2) \\ \vdots & \vdots \\ \bar{S}_p(T) & \bar{C}_p(T) \end{bmatrix} \]

- Dynamic sinogram:

\[ \bar{y} = [\bar{y}'_1 \ \bar{y}'_2 \ldots \ \bar{y}'_T]' = \left( A \bigotimes P \right) \begin{bmatrix} \kappa_i \\ b \end{bmatrix} + \mathbf{r}, \]

- Log-likelihood function:

\[ L(y|x) = \sum_{n=1}^{T} L(y_n|x_n) = \sum_{n=1}^{T} \sum_{i=1}^{M} (y_n)_i \log(\bar{y}_n)_i - (\bar{y}_n)_i \]

Or, \[ L(y|\kappa_i, b) = \sum_{n=1}^{T} \sum_{i=1}^{M} (y_n)_i \log(\bar{y}_n)_i - (\bar{y}_n)_i \]
Dynamic PML Reconstruction

• Indirect reconstruction

Step 1: \[ \hat{x}_n(y_n) = \arg \max_{x_n \geq 0} [L(y_n|x_n) - \beta_x \phi(x_n)] \]

Step 2: \[ \begin{bmatrix} \hat{\kappa}_i \\ \hat{b} \end{bmatrix} = \arg \min_{\kappa_i, b \geq 0} \left\| \hat{x} - (A \otimes I_N) \begin{bmatrix} \kappa_i \\ b \end{bmatrix} \right\|^2 \quad \text{(Indirect1)} \]

Alternatively, Indirect2:

\[ \begin{bmatrix} \hat{\kappa}_i \\ \hat{b} \end{bmatrix} = \arg \min_{\kappa_i, b \geq 0} \left[ \frac{1}{2} \left\| \hat{x} - (A \otimes I_N) \begin{bmatrix} \kappa_i \\ b \end{bmatrix} \right\|^2 + \beta_{\kappa_i} \phi(\kappa_i) + \beta_b \phi(b) \right] \]

• Direct reconstruction

\[ \begin{bmatrix} \hat{\kappa}_i \\ \hat{b} \end{bmatrix} = \arg \max_{\kappa_i, b \geq 0} [L(y|\kappa_i, b) - \beta_{\kappa_i} \phi(\kappa_i) - \beta_b \phi(b)] \]
SNR of Indirect1 Reconstruction

- Apply CHO on $\kappa_i$ and figure of merit
  \[
  \text{SNR}_{\text{CHO}}^2 = z'U'K^{-1}Uz
  \]

- Closed-form solution of Indirect1
  \[
  \begin{bmatrix}
  \hat{\kappa}_i \\
  \hat{b}
  \end{bmatrix} = [(A' A)^{-1} A' \otimes I_N] \hat{x}
  \]

- Mean and covariance of dynamic image $x$
  \[
  z_{\hat{x}}(\beta_x) = \begin{bmatrix}
  z_{\hat{x}_1} \\
  \vdots \\
  z_{\hat{x}_T}
  \end{bmatrix}, \quad \Sigma_{z_{\hat{x}}}(\beta_x) = \begin{bmatrix}
  \Sigma_{\hat{x}_1} & \cdots \\
  \vdots & \ddots \\
  \Sigma_{\hat{x}_T}
  \end{bmatrix}
  \]

- Mean and covariance of $\kappa_i$
  \[
  \begin{align*}
  z_{\kappa_i, b}^{\text{Indirect1}} &= [(A' A)^{-1} A' \otimes I_N] z_{\hat{x}}, \\
  \Sigma_{\kappa_i, b}^{\text{Indirect1}} &= [(A' A)^{-1} A' \otimes I_N] \Sigma_{z_{\hat{x}}} [(A' A)^{-1} A' \otimes I_N]',
  \end{align*}
  \]
  \[
  \text{SNR}_{\text{CHO}}^{\text{Indirect1}}(\beta_x)
  \]
SNR of Indirect2 Reconstruction

• Quadratic penalty

\[ \beta_{\kappa_i} \phi(\kappa_i) + \beta_b \phi(b) = \frac{1}{2} \left[ \begin{array}{c} \kappa_i \\ b \end{array} \right]' \left[ \begin{array}{cc} \beta_{\kappa_i} R & \beta_b R \\ \beta_{\kappa_i} R & \beta_b R \end{array} \right] \left[ \begin{array}{c} \kappa_i \\ b \end{array} \right] \]

• Closed-form solution of Indirect2

\[
\begin{bmatrix} \hat{\kappa}_i \\ \hat{b} \end{bmatrix} = \left( A' A \otimes I_N + R_d \right)^{-1} (A \otimes I_N)' x = Bx,
\]

• Mean and covariance of \( \kappa_i \)

\[
\begin{align*}
\mathbb{E}^{\text{Indirect2}}(\hat{\kappa}_i, \hat{b}) &= B \mathbb{E}^{\hat{x}}, \\
\mathbb{E}^{\text{Indirect2}}(\hat{\kappa}_i, \hat{b}) &= B \mathbb{E}^{\hat{x}} B'
\end{align*}
\]

\[ \text{SNR}^{\text{Indirect2}}(\beta_x, \beta_{\kappa_i}, \beta_b) \]
SNR of Direct Reconstruction

- Considering $A \otimes P$ as a single system matrix
- Fisher information matrix
  \[ F = (A \otimes P)' \text{diag}[\frac{1}{y}] (A \otimes P) \]
- Quadratic penalty
  \[ \beta_{\kappa_i} \phi(\kappa_i) + \beta_b \phi(b) = \frac{1}{2} \begin{bmatrix} \kappa_i \\ b \end{bmatrix}' \begin{bmatrix} \beta_{\kappa_i} R \\ \beta_b R \end{bmatrix} \begin{bmatrix} \kappa_i \\ b \end{bmatrix} \]
  \[ = \frac{1}{2} \begin{bmatrix} \kappa_i \\ b \end{bmatrix}' R_d \begin{bmatrix} \kappa_i \\ b \end{bmatrix} \]
- Mean and covariance of $\kappa_i$
  \[ z_{\hat{\kappa}_i, b}^{Direct} = (F + R_d)^{-1} F \hat{f}_{\kappa b}, \]
  \[ \Sigma_{\hat{\kappa}_i, b}^{Direct} = (F + R_d)^{-1} F (F + R_d)^{-1} \]
  \[ \text{SNR}_{\text{CHO}}^{Direct}(\beta_{\kappa_i}, \beta_b) \]

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Fast Computation

• Using Taylor series and locally shift-invariant approximation

Fast evaluate lesion detectability and then guide the selection of $\beta$ for each method
Simulation Method

• GE 690 PET/CT scanner
• A 2D digital phantom
• TACs
  • Extracted from one patient
  • 60-minute scan, 49 frames
  • 24 million total counts
Theoretical SNRs

- **Static:** $\text{SNR}(\beta_x)$
- **Indirect1:** $\text{SNR}(\beta_x)$
- **Indirect2:**
  \[ \text{SNR}(\beta_x) = \arg\max_{\beta_{\kappa_i}, \beta_b} \text{SNR}(\beta_x, \beta_{\kappa_i}, \beta_b) \]
- **Direct:**
  \[ \text{SNR}(\beta_{\kappa_i}) = \arg\max_{\beta_b} \text{SNR}(\beta_{\kappa_i}, \beta_b) \]
Theoretical vs Monte Carlo SNR

- Monte Carlo SNR was computed from 200 noisy realizations

SNR of theoretical (curves) and Monte Carlo results ("x", "△", "○", "□")
Sample Reconstructions

(Each reconstruction method with the corresponding optimal penalty parameters)
SNR under Different TACs

- Extract breast tumor TACs from other three patients
Shortened Protocol

- One limitation of dynamic PET: prolonged scan time
- Dual-time Patlak reconstructions for whole-body dynamic PET (Karakatsanis et al 2013a, 2013b, Zhu et al 2014)
- Theoretical SNR

![Graph showing SNR comparison]

7 frames: 25 min – 60 min
2 frames: 25 min – 30 min & 55 min – 60 min
Clinical Implementation

Data acquisition

Dynamic Sinograms \{y\}

Frame-by-frame OSEM reconstruction

Dynamic Images \{x\}

Indirect 2 reconstruction

Regularization optimization: maxSNR

Parametric Images \{\kappa_i, b\}

Extract TACs:
- Blood input function
- Tumors
- Background organs

Scan time (min)

Activity

Scan time (min)
Reconstructed Images

Static

Indirect1

Indirect2

(Each reconstruction method with the corresponding optimal penalty parameters)
Reconstructed Images

Static  Indirect1  Indirect2

(Each reconstruction method with the corresponding optimal penalty parameters)
Summary

• We have developed a method to evaluate and optimize PML reconstruction for lesion detection from static PET to dynamic PET

• The benefit of static task-based PML reconstruction (Resolution Modeling + Penalty Design) was demonstrated using computer-based Monte Carlo simulation and real data

• Extension to dynamic PET showed further improvement for lesion detection
Future Work

- Shift-variant resolution modeling
- Analysis of dynamic PET reconstruction based on non-linear kinetic model
- Extension to localization task and quantification task
- Extension to the next generation of PET scanner, such as EXPLORER
  - Penalty design for lesion detection
  - Whole-body dynamic PET analysis
Publication

Journal:


Peer-reviewed conference paper:


Peer-reviewed conference abstract:


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